Space Charge Effects During Bunching

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Longitudinal space charge in the proton driver can strongly perturb the bunch rotation that seems required to produce short proton bunches for the muon collider. The longitudinal electric field produced by the space charge is given by

$$E = -\frac{g_0 \lambda'}{4\pi\varepsilon_0 \gamma^2},$$

where $g_0 = (1+2\ln(b/a))$, b is the vacuum chamber radius and a is the beam radius λ' is the charge density gradient, ε_0 is the permittivity of free space and γ is the relativistic factor. The voltage produced on a particle in one turn is

$$V_i = -\frac{Rg_0\lambda'}{2\varepsilon_0\gamma^2},$$

where $2\pi R$ is the circumference of the ring. As the bunch is compressed over a N turns, an individual particle will lose (or gain) an amount of energy, shown in Fig 1, given by

$$U = q\sum_{i=1}^{N} V_i = q\langle V\rangle N = q\langle V\rangle \frac{\Delta\phi_{\max}}{2\pi\eta\,\Delta p/p},$$

where $\langle V\rangle$ is the simple average of the space charge induced voltage per turn during the compression process, q is the charge and U must be in units like W introduced below. The value of synchrotron phase at the start of the compression is $\Delta\phi_{\max}$, and the expression in the denominator is the phase change per turn for a given slip factor η and momentum offset, $\Delta p/p$, since $\delta\phi_{turn}/2\pi = \delta f/f = \eta\,\Delta p/p$.

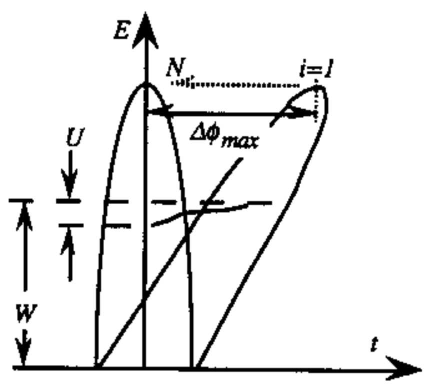

Figure 1. Nonlinear effects of phase shear.

The complete expression can be written more explicitly in terms of the average charge density gradient $\langle\lambda'\rangle$ rather than the average voltage per turn $\langle V\rangle$

$$U = \frac{qRg_0\langle\lambda'\rangle\Delta\phi_{\max}}{4\pi\varepsilon_0\gamma^2\eta\,\Delta p/p},$$

and it is possible to get the beam energy γ to appear only once by using the expression

$$\gamma^2\eta = \gamma^2\left(1/\gamma^2 - 1/\gamma_t^2\right) = \left(1-\gamma^2/\gamma_t^2\right).$$

This gives

$$U = \frac{qRg_0\langle\lambda'\rangle\Delta\phi_{\max}}{4\pi\varepsilon_0(1-\gamma^2/\gamma_t^2)\,\Delta p/p}.$$

As long as the energy loss by the particle is much less than the initial energy displacement from the center of the bunch, W, distortion of the bunch will be minimal. Thus the condition for bunching is that

$$\frac{U}{W} = \frac{qRg_0\langle\lambda'\rangle\Delta\phi_{\max}}{4\pi\varepsilon_0 W(1-\gamma^2/\gamma_t^2)\,\Delta p/p} < 1.$$

Values for the parameters can be obtained numerically for the AGS and the driver of the muon collider for a bunch compression using widths from FWHM / 2 = 10 ns down to 1 ns. Assuming a triangular bunch for simplicity, the values of V_i and $\langle V\rangle$ vs the full width, look like Fig 2. The voltage per turn during bunching is useful because it can be compared with calculations for specific cases. Note that $V_i \sim$ width^{-2}.

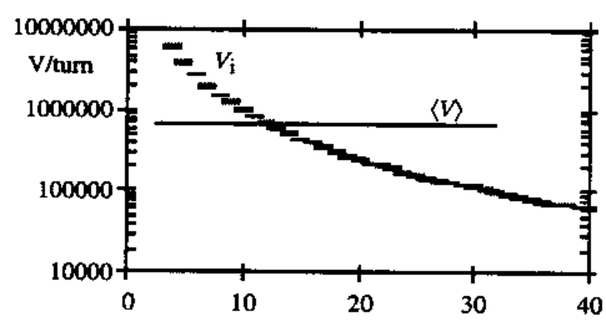

Figure 2. V_i and $\langle V\rangle$ plotted vs. bunch full width for a triangular bunch in the AGS.

Using these approximations it is possible to calculate ratios U/W for a given ratio of beam and transition energies. Note that since only the energy ratio appears, a lattice with an adjustable γ_t can minimize the longitudinal space charge effects at any energy by setting the transition energy as far from γ as possible. The parameters can be summarized in the table below and the results are shown in Fig 3.

Table I. Bunch Parameters

	AGS	$\mu\mu$ Driver	
Charge	$0.8 \cdot 10^{13}$	$2.5 \cdot 10^{13}$	p/b
I_p	640	2000	A
γ	8	15	
$2\pi R$	808	600	m
$\langle V \rangle$	0.678	0.448	MV
$\langle \lambda' \rangle$	$4.0 \cdot 10^{-7}$	$1.25 \cdot 10^{-6}$	C/m^2

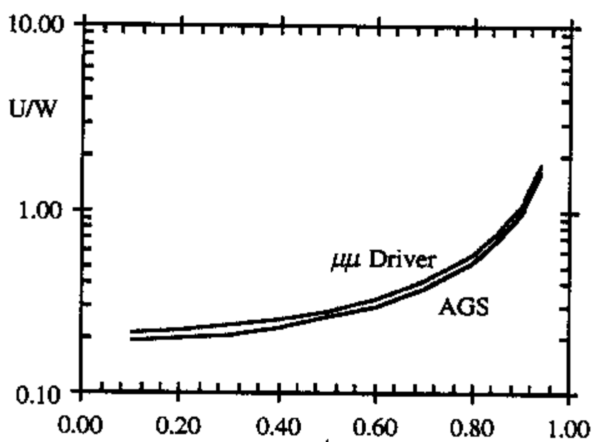

Figure 3. Nonlinearity vs γ/γ_t

A plot of the energy loss U against time or number of turns during the final bunching shows that the last two or three turns contribute most of the energy loss, see Figure 4, thus bunching will proceed almost unaffected by space charge until the process is almost complete.

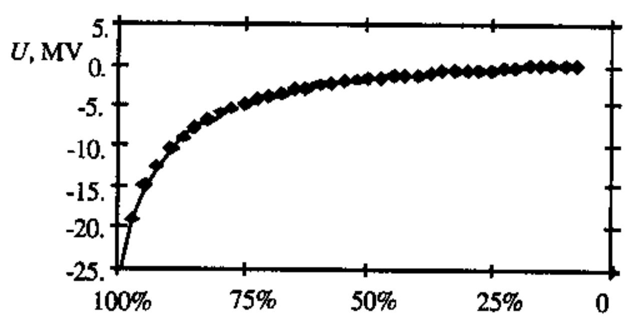

Figure 4, Energy loss plotted against the completeness of the final bunch rotation.

Since the design of the accelerator enters into this expression as $R/W(1 - \gamma^2/\gamma_t^2)\Delta p/p$, the size of the ring should be minimized, likewise the momentum aperture and the difference between the operating and transition energy should be maximized. (Because the energy offset $W \approx \Delta p$, the momentum spread essentially enters with an exponent of 2). Thus one wants a lattice with an efficient packing factor for dipoles, magnets with a large aperture and the transition energy as far from the beam energy as possible during the final bunch rotation.

Space charge effects during bunching seem to impose no restrictions on the machine energy if the transition energy is regarded as a free parameter, nevertheless instabilities due to other effects may very well be energy dependent

This calculation assumes that the bunching is done below transition, if the bunching were done above transition the calculation is the same, except the bunch would move from the left to the center on Figure 1, and the particles would gain energy rather than lose energy. One would still want U/W to be much less than one, but the problem then would be scraping on the momentum aperture of the machine rather than the loss of momentum spread which would prevent bunching.

The use of the triangular bunch approximation requires some justification, however the bunch shape seems at least as good as any other simple parameterization. Measurements of the shape of an adiabatically captured bunch in the Fermilab Booster near transition show a bunch that is quite triangular, with very slightly rounded peak and corners (Fig. 5). (The secondary peak is a reflection.) In principle it is possible to control the bunch shape to optimize bunching, and a triangular bunch would be optimum.

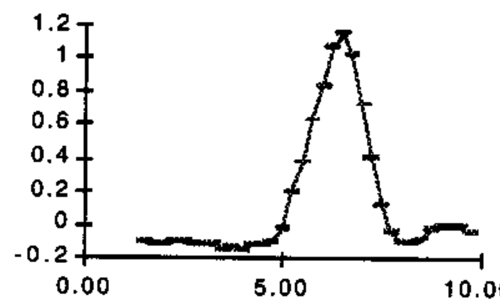

Figure 5. Bunches in the Fermilab Booster.

The effects discussed above are independent of the method of bunching, and the methodology and conclusions should apply to all. For example, if an energy shear is done near γ_t, followed by a shear in phase, far from γ_t, the transition energy should be moved roughly 30% higher or lower to permit the bunching to be fast enough to overpower space charge effects. Likewise if the bunch is stretched by lowering V_{rf}, followed by a rotation either above or below transition, the beam energy must be different from γ_t by the same factor.

FMC lattices seem to have the capability of moving the transition energy by a sufficient amount to permit consideration of these methods.